

GEOMETRICALLY DEVELOPED STOCHASTIC DYNAMICS AND APPLICATIONS

Debasish Roy¹

¹ *Computational Mechanics Lab, Department of Civil Engineering, Convener, Centre of Excellence in Advanced Mechanics of Materials, Indian Institute of Science, Bangalore, India, royd@iisc.ac.in*

ABSTRACT

In this talk, we consider and explore a method for developing the flows of stochastic dynamical systems, posed as Ito's stochastic differential equations, on a Riemannian manifold identified through a suitably constructed metric. The framework used for the stochastic development, viz. an orthonormal frame bundle that relates a vector on the tangent space of the manifold to its counterpart in the Euclidean space of the same dimension, is the same as that used for developing a standard Brownian motion on the manifold. Mainly drawing upon some aspects of the energetics so as to constrain the flow according to any known or prescribed conditions, we show how to expediently arrive at a suitable metric, thus briefly demonstrating the application of the method to a broad range of problems of general scientific interest. These include simulations of Brownian dynamics trapped in a potential well, a numerical integration scheme that reproduces the linear increase in the mean energy of conservative dynamical systems under additive noise and non-convex optimization. Of specific interest in this talk is our geometrically inspired approach for non-convex optimization problems, wherein we exploit stochastically developed Langevin dynamics with and without multiplicative (state-dependent) noises. The simplicity of the method and the sharp contrast in its performance vis-à-vis the correspondent Euclidean schemes in our numerical work provide a compelling evidence to its potential.

AXISYMMETRIC VIBRATIONS OF SOFT ELECTROACTIVE CYLINDRICAL SHELLS

Fangzhou Zhu¹, Bin Wu, Weiqiu Chen¹

¹Key Laboratory of Soft Machines and Smart Devices of Zhejiang Province and Department of Engineering Mechanics, Zhejiang University, Hangzhou 310027, P.R. China, chenwq@zju.edu.cn

ABSTRACT

Soft electroactive (SEA) materials exhibit the exotic capability of high-speed electrical actuation with strains greater than 100%. In addition, they also possess many other excellent electromechanical properties such as low actuation voltage, high fracture toughness and energy density. They therefore have received considerable academic and industrial interest, and found widespread applications ranging from actuators, sensors and energy harvesters to biomedical and flexible electronic devices. However, there are three main difficulties associated with soft electroactive (SEA) structures, i.e. coupling between electric and mechanical field, nonlinear material constitutive behavior and large deformation. Here we report a study on axisymmetric vibrations of an incompressible SEA cylindrical shell. Either axisymmetric torsional or longitudinal vibrations are considered when the cylindrical shell is subject to an inhomogeneous biasing field, which is induced by radial electric voltage and axial pre-stretch. The state-space method is employed to derive the frequency equations for two separate classes of axisymmetric vibration of the cylindrical shell. Numerical examples are considered to validate the convergence and accuracy of the method. The results demonstrate that the axisymmetric vibration characteristics of the SEA cylindrical shell could be manipulated significantly by properly choosing the electromechanical biasing field as well as its geometry.

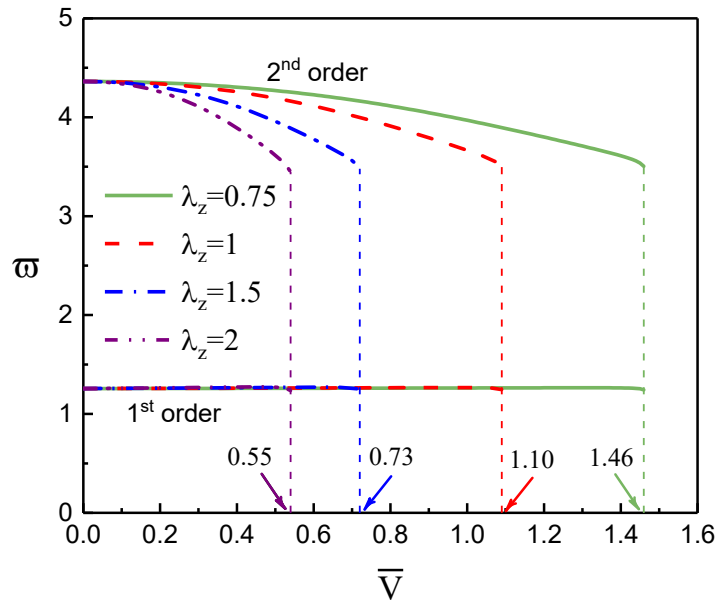


Figure 1: The first two resonant frequencies ω of the torsional vibration with $n=1$ as functions of the radial electric voltage \bar{V} for a thick and short SEA cylindrical shell under different axial pre-stretches λ_z

Keywords: Axisymmetric vibration, Soft electroactive material, Cylindrical shell

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THE PARAMETER SPECTRAL ANALYSIS THEORY AND THE GENERALIZED CENTRAL DIFFERENCE METHOD

Yufeng Xing¹, Yi Ji²

¹ Institute of Solid Mechanics, Beihang Univeristy, China, xingyf@buaa.edu.cn

² Institute of Solid Mechanics, Beihang Univeristy, China, jiyi0319@buaa.edu.cn

ABSTRACT

For examining the stability of a time integration method when it is used to solve nonlinear dynamics, we have proposed a parameter spectral analysis method, of which the main idea is the introduction of a scalar δ_t , the ratio between the frequency ω_t of the current time point and the frequency ω_{t-h} of the previous point. The used single degree-of-freedom system in this spectral method is

$$\ddot{\mathbf{x}}_t + 2\xi\delta_t\omega_{t-h}\dot{\mathbf{x}}_t + (\delta_t\omega_{t-h})^2\mathbf{x}_t = 0$$

This theory can find stability conditions of time integration methods, see Table 1. Numerical experiments indicate that these stability conditions can accurately capture the moment at which time integration methods begin to diverge.

Table 1: Stabilities of some time integration methods.

Method	Formulations	Stability criteria	Stability Limit ($\delta_t=1$)
CDM	$\ddot{\mathbf{x}}_t = (\mathbf{x}_{t-h} - 2\mathbf{x}_t + \mathbf{x}_{t+h})/h^2$ $\dot{\mathbf{x}}_t = (-\mathbf{x}_{t-h} + \mathbf{x}_{t+h})/2h$	$\delta_t^2 \tau_{t-h}^2 - 4 \leq 0$ or $\tau_t^2 - 4 \leq 0$	$\tau^2 - 4 \leq 0$
EG- α , $\rho_\nu=0$ [1]	$\mathbf{x}_{t+h} = \mathbf{x}_t + h\dot{\mathbf{x}}_t + h^2(-2\ddot{\mathbf{x}}_t + 5/2\ddot{\mathbf{x}}_{t+h})$ $\dot{\mathbf{x}}_{t+h} = \dot{\mathbf{x}}_t + h(-3/2\ddot{\mathbf{x}}_t + 5/2\ddot{\mathbf{x}}_{t+h})$	$\delta_t^2 \tau_{t-h}^2 - 12/5 \leq 0$ or $\tau_t^2 - 12/5 \leq 0$	$\tau^2 - 12/5 \leq 0$
TR	$\mathbf{x}_{t+h} = \mathbf{x}_t + h\dot{\mathbf{x}}_t + h^2(\ddot{\mathbf{x}}_t + \ddot{\mathbf{x}}_{t+h})/4$ $\dot{\mathbf{x}}_{t+h} = \dot{\mathbf{x}}_t + h(\ddot{\mathbf{x}}_t + \ddot{\mathbf{x}}_{t+h})/2$	$1 - \delta_t^2 \leq 0$ or $\Delta_t < 0$	Unconditionally stable
CH- α [2]	$\mathbf{x}_{t+h} = \mathbf{x}_t + (\mathbf{I} + 1/2h^2\mathbf{M}^{-1}\mathbf{K}_0)^{-1}h\dot{\mathbf{x}}_t + 1/2(\mathbf{I} + 1/2h^2\mathbf{M}^{-1}\mathbf{K}_0)^{-1}h^2\ddot{\mathbf{x}}_t$ $\dot{\mathbf{x}}_{t+h} = \dot{\mathbf{x}}_t + h(\ddot{\mathbf{x}}_t + \ddot{\mathbf{x}}_{t+h})/2$	$\tau_{t-h}^2(1 + \delta_t^2) - 4(2 + \tau_0^2) \leq 0$	Unconditionally stable

In addition, using the parameter spectral analysis method, we have developed a generalized central difference method, as

$$\ddot{\mathbf{x}}_t = (\mathbf{x}_{t-h} - 2\mathbf{x}_t + \mathbf{x}_{t+h})/h^2, \dot{\mathbf{x}}_t = (\mathbf{x}_{t+h} - \mathbf{x}_{t-h})/(2h), \mathbf{x}_t = (\mathbf{x}_{t+h} + 2\rho_\infty\mathbf{x}_t + \rho_\infty^2\mathbf{x}_{t-h})/(1 + \rho_\infty)^2 \text{ and } \mathbf{M}\ddot{\mathbf{x}}_t + \mathbf{N}(\mathbf{x}_t, \dot{\mathbf{x}}_t) = \mathbf{R}_t,$$

which is unconditionally stable for damped and undamped ($\xi=0$) nonlinear systems, as shown in Fig. 1. Another important thing is that the algorithmic parameters of this new time integration method are determined by optimizing low-frequency accuracy. Numerical experiments demonstrate the stability, accuracy and efficiency advantages of this time integration method over other methods.

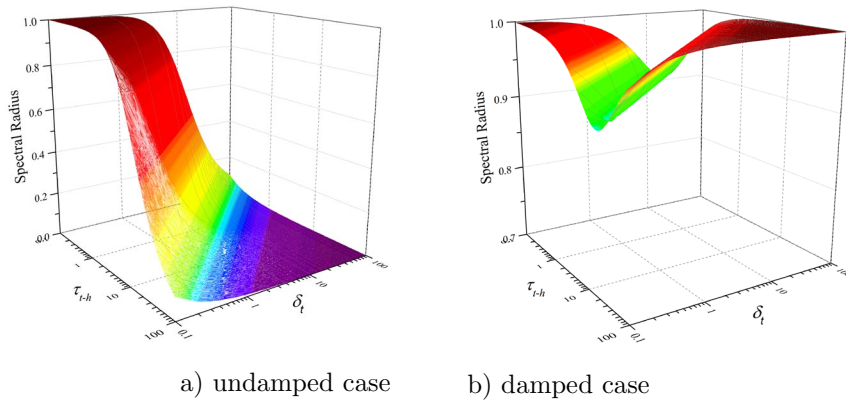


Figure 1: Spectral radius versus δ_t and τ_{t-h} : a) undamped case ($\rho_\infty=0$, $\xi=0$); b) damped case ($\rho_\infty=1$, $\xi=0.1$)

Keywords: Nonlinear system, Parameter spectral analysis, Time integration methods, Unconditional stability; Optimization

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NONLINEAR VIBRATIONS IN BIO-ELECTRO-MECHANICAL SYSTEM OF THE HUMAN MIDDLE EAR

Rafal Rusinek¹, Andrzej Weremczuk², Krzysztof Kecik³

¹ Lublin University of Technology, Department of Applied Mechanics, Nadbystrzycka 36, 20-618 Lublin, Poland, e-mail: r.rusinek@pollub.pl

² Lublin University of Technology, Department of Applied Mechanics, Nadbystrzycka 36, 20-618 Lublin, Poland, e-mail: a.weremczuk@pollub.pl

³ Lublin University of Technology, Department of Applied Mechanics, Nadbystrzycka 36, 20-618 Lublin, Poland, e-mail: k.kecik@pollub.pl

ABSTRACT

The middle ear is one of the smallest biomechanical systems in the human body. Therefore a treatment of the ear is especially demanding task. An implantable middle ear hearing device (IMEHD) is one of the technique, used in clinical practice, to improve the hearing process [1]. To investigate the IMEHD, a 6-degree of freedom (6dof) model is proposed here. It is composed of 3dof subsystem of the middle ear, 2dof subsystem of the IMEHD and excitation current flow through the transducer. That gives us 6dof model of the implantable middle ear (Fig.1) which is nonlinear both for the sake of properties of the middle ear and the IMEHD. The main objective of this paper is to explain the role of both (a) the electromechanical coupling between the mechanical and the electromagnetic subsystems and (b) properties of an implant clip, which connects the stapes to a main part of the implant - the floating mass transducer. The proposed bio-electro-mechanical model should generate interesting nonlinear phenomena, especially when excitation and coupler stiffness change. To find resonance curves and bifurcation diagrams the method of multiple scales is engaged. As a result, different types of system response are observed, also on the basis of numerical simulations. Results obtained in this study can be used to formulate recommendations for practical implementation.

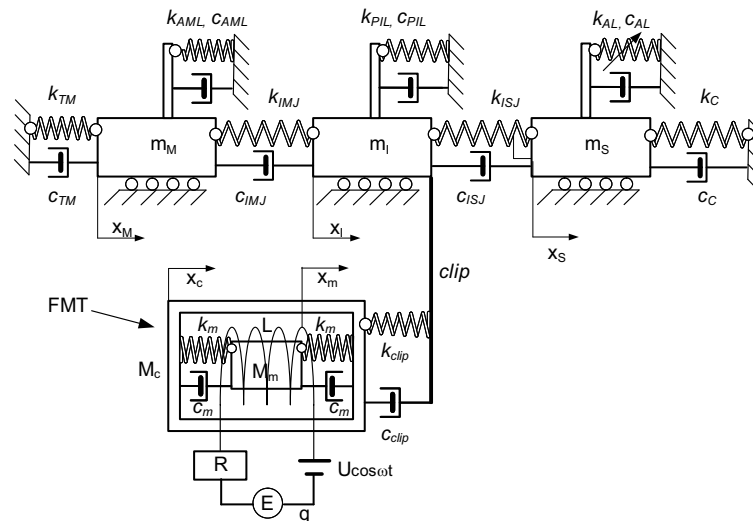


Figure 1: Six degree of freedom model of the implanted middle ear.

Keywords: Middle ear, Floating mass transducer, Ear implant

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SD OSCILLATORS, ISOLATORS AND SUSPENSOR FREE OF RESONANCE

Qingjie Cao

School of Astronautics, Harbin Institute of Technology, Harbin 150001 China, Email. Q.j.cao@hit.edu.cn

ABSTRACT

Mechanical resonance phenomenon, seen in [1], was discovered by Euler L. in 1750 when he investigated the forced mass-spring system. Mechanical resonance has been a huge obstacle since the last century to a wide range of modern engineering. In this talk we present the steps to remove the engineering resonances, as the following.

1. **SD Oscillators.** Starting from a single SD oscillator, seen in [2], with negative stiffness (SNS), seen in [3], the k -multiple SD oscillators or (k -MNS) ($k = 1, 2, \dots$) can be constructed by linking k SD oscillators to the mass with the other ends pinned to each fixed points separately providing k geometrical parameters.
2. **QZS Isolators.** The quasi-zero stiffness (QZS) isolators, seen in [4], of order $2k - 1$ can be integrated by connecting the conventional harmonic positive oscillator in parallel with the k -MNS unit.
3. **Suspensor.** Taking the limit of the high-order QZS systems, we reach the so called **mechanical suspensor** free of resonance within a prescribed distance to nullify gravity by relative mass enabling human beings to experience the space life with zero gravity on earth and to live aboard spaceship with ground gravity.

Isolation performance of linear, QZS system and the suspensor has been obtained theoretically, shown in Fig.1(a), and the performance of the suspensor has also been tested experimentally, shown in Fig.1 (b) and (c) for isolation, (d) and (e) for impact, the details seen in the corresponding captions.

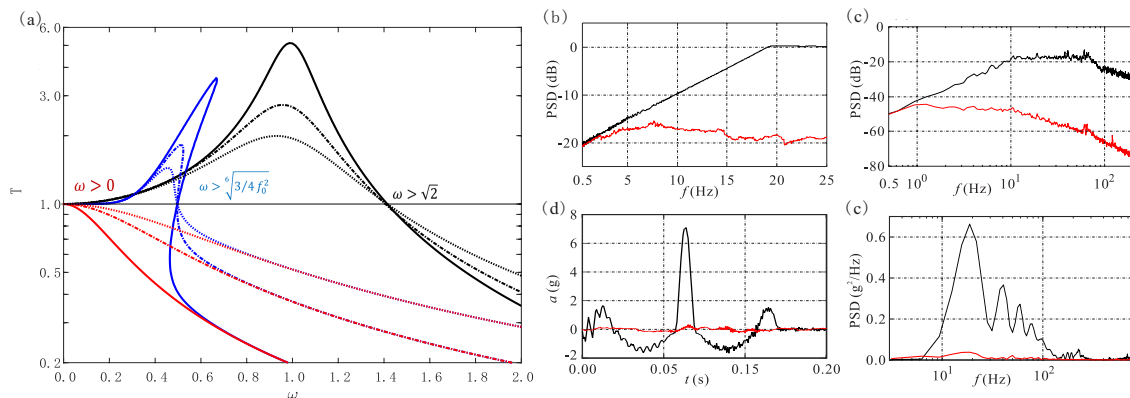


Figure 1: (a) Transmissibility for linear, QZS and suspensor for black, blue and red with solid, dashed and dotted for damping ratio $\xi = 0.1, 0.2$ and 0.3 , respectively, (b) and (c) the sweeping and random frequency, (d) and (e) the impact responses for time and frequency domain, the black and the red for input and responses, respectively.

Keywords: SD oscillators, High-order QZS isolators, Suspensor, Mechanical resonance, Free of resonance

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