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VIBRATION OF A CANTILEVER IN A MICRO-CHANNEL INTERACTING WITH RAREFIED GAS FLOW.

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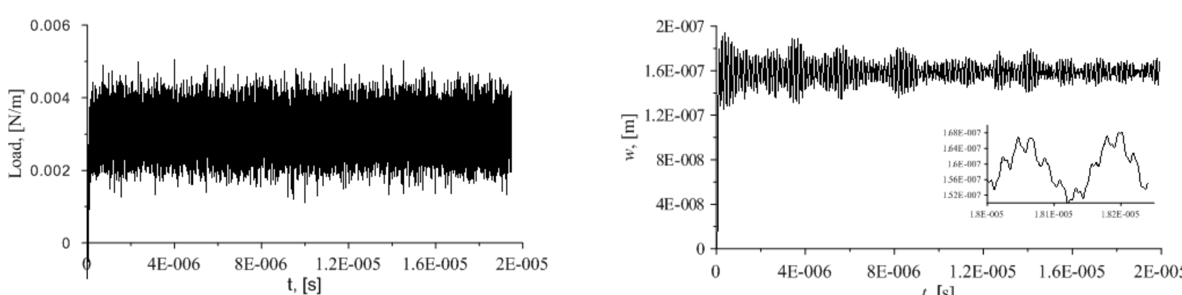
ABSTRACT

The analyses of gas flow at microscale level are important tasks for many high technological devices and they are subjects of studies of many researchers. The essential and intriguing element is that there are fundamental differences in the microfluidic considerations with respect to those of conventional flows at macroscales. In many cases in micro channels, where the gas flows, there are obstacles with different shapes. In most of the cases they are considered as rigid bodies. The proper model of such system requires considering the deformability of the obstacle. The consideration of the elastic properties of the obstacle means that it will deform due to the gas flow and due to its motion, the gas flow will be disturbed.

In the authors' previous work [1] a numerical approach and computer code are developed to study 2D rarefied gas flow in a microchannel having an elastic obstacle. The gas flow is simulated by the Direct Simulation Monte Carlo (DSMC) method applying the advanced Simplified Bernoulli Trial (SBT) collision scheme. The beam vibration is modeled by a reduced 3-modes model of the Euler Bernoulli beam.

The nonlinear system of ordinary differential equations was solved by using an implicit numerical scheme in the time domain. The numerical studies were performed for a fixed Knudsen number and different flow velocities.
The goal of this work is to extend our previous work. The computer code is parallelized in order to allow studying the influence of the Knudesn number and the velocity of the gas on the vibration of the beam. The response of the beam during its interaction with the gas at Mach number M=1.6 and Knudsen number Kn=0.05 is presented in Fig. 1 a, b. The stochastic nature of the DSMC and the way of the consideration of the interaction between the gas particles and the beam lead to a stochastic character of the loading. In spite of the chaotic character of the loading three frequencies that correspond to the first natural frequencies dominate the response of the beam. After a transition period, the elastic beam vibrates with very small amplitudes around a new equilibrium state (bended). The approximate amplitudes of the beam vibration at a time close to the final bended state of the beam can be seen at the small figure inserted in the Fig. 1 (b). The influence of the geometrical nonlinearity of the beam on the response of the beam at different Knudsen number is analyzed and the responses are compared with ones of the linear beam vibration.



(a) (b)

Figure 1: The loading (a), time-history diagram (b) and frequency-response functions at the tip of the beam for M=1.6

Keywords: rarefied gas-flow, microchannel, DSMC, Euler Bernoulli beam.

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NONLINEAR BEAM APPROXIMATIONS TO THE PRINCIPLE OF VIRTUAL WORK

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ABSTRACT

Over a considerable period of time, the author has been developing and improving a unique numerical, finite-element based technology for the analysis of slender beams undergoing arbitrary large displacements and rotations, yet with small strains, in three-dimensional space, and with the application of time-varying end displacements and loadings. The application to industry has been focused on the response analysis of riser systems connected to floating production platforms in the offshore environment ranging from shallow water to ocean depths beyond 3,000 metres. Key references for this analysis technology are [1, 2, 3].

A key challenge in the development of beam mechanics is to effectively one-dimensionalise the Principle of Virtual Work equation; essentially a slender beam has the one dimension of distance along its neutral axis, with the beam cross-section terms incorporated through an integral over the cross-section resulting in generalized stresses such as axial force, shear forces, bending moments and torque defined at each point along the neutral axis of the beam. Herein, lies some difficulty in correctly determining that one-dimensional virtual work equation in order to include all response effects. Approximations are always being made to simplify the problem and to provide practical solutions to categorized problems, although the nature or implications of such approximations has not always been made clear.

The approach adopted in this paper is grounded in the following fundamentals:

1. The definition of Green's strain as a material strain, written in material coordinates and its precise link to the virtual strain term in the Principal of Virtual Work.
2. The various tensor representations of stress and the invariance of the inner stress-strain tensor product in the Principal of Virtual Work. Vector and co-vector spaces are utilized to establish the correct invariant term while highlighting the exact nature of the "conjugate stress" terms relative to Green's strain for the deformed beam.
3. The derivation of the equilibrium equations relative to a unique convected co-ordinate system developed previously by the author [1,2] using Euler Angles with approximations that are appropriate for moderate beam rotations relative to this convected axes.

The above derivations make extensive use of generalized coordinate systems, coordinate transformations and associated tensor transformations.

The work will clearly highlight the range of approximations that can be made in the development of beam mechanics and the implications of such approximations for the solution of three-dimensional, large displacement beam problems. The paper will provide a precise insight on the nature of the various terms in the beam Principle of Virtual Work equation.

Keywords: Beam Mechanics, Large three-dimensional rotations, Principle of Virtual Work, Conjugate Stress and Strain, Convected Coordinate Axes

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ROLE OF ELECTROMECHANICAL COUPLING IN THE VIBRATION ABSORBER/HARVESTER SYSTEM

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ABSTRACT

The tuned mass absorber is widely used for passive vibration control in various engineering structures. When it is attached to the primary (main) system, the unwanted response can be suppressed. The additional harvester device mounted in the vibration absorber allows simultaneous vibration control and energy recovery [1]. The dynamic system is known as the absorber/harvester [2]. The one main vibration energy harvesting technology is electromagnetic. It generates power from changes in magnetic field due to motion of the magnet vs. coil winding. However, there are a number of difficulties associated with modelling of such systems. The first is how the energy harvester added to the vibration absorber influences vibration mitigation effectiveness. The second is the way of electromechanical coupling modelling.

This paper discusses the analysis of the four degree of freedom system. It consists of the pendulum vibration absorber (Fig. 1a) and the electromagnetic harvester device (Fig. 1b) with the circuit (Fig. 1c). This system is strongly nonlinear can exhibit different behaviors. The frequency-amplitude responses and induced current in the coil are investigated in detail. The various models of electromechanical couplings (linear and nonlinear) are proposed. Results obtained in this study can be used to improving efficiency of the absorber/harvester system to harvest energy without a loss in the vibration mitigation.

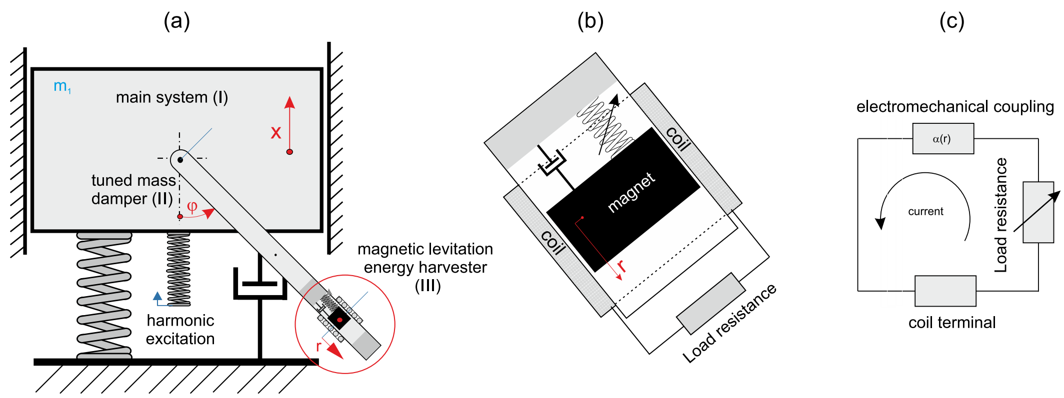

Figure 1: The vibration absorber/harvester model (a), the harvester device (b) and the electrical circuit (c).

Keywords: Electromechanical coupling, Energy harvesting, Vibration mitigation

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ENERGY HARVESTING FROM A ROTATING PARAMETRIC PENDULUM: SINGULAR OPTIMAL CONTROL

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ABSTRACT

Sea waves represents a very promising energy source. Pioneered by Wiercigroch a pendulum system would be feasible for such an energy harvesting purpose (see for instance[1] and [2]).

Consisting of a pendulum with a vertical motion induced by the sea waves, pendulum's stable rotations generate enough energy to be extracted by an electrical generator attached to its axis [2].

In this paper, a brush-less dc motor (control input $u(t)$) will provide stable pendulum's rotations. The mechanical model is given by equation (1):

$$\ddot{\theta}(t) + \beta \cdot \dot{\theta}(t) + \left(R \cdot cos(\omega \cdot t) + \lambda \cdot R \cdot \frac{\Lambda_3}{\Lambda_1^3} + \lambda \cdot \frac{\Lambda_2}{\Lambda_1} + 1 \right) + u(t) = 0 \tag{1}$$

Following the ideas implemented in [3], a singular optimal control formalism provides a very simple control law in equation (2) rewriting equation (1) in state-space form :

$$min_{u \in U} \quad \frac{1}{2} \left(\dot{\theta}(t) - \phi(t) \right)^2 \tag{2}$$

such that:

$$\dot{X}(t) = \begin{bmatrix} x_2(t) \\ h(x_1, x_2, t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot u(t)$$

where $X(t) = [x_1, x_2]', x_1 = \theta(t), x_2(t) = \dot{\theta}(t)$ and $\phi(t)$ acts as a set-point (desired pendulum's trajectory). Notice that tuning the function $\phi(t)$, different controlled behaviours for the pendulum's orbits can be achieved.

Pontryagin's principle solve this singular optimal control problem in equation (3):

$$u(t) = -K \cdot sign \left(\dot{\theta}(t) - \phi(t) \right) \tag{3}$$

with $K \in \Re^+$ and arbitrary constant and $sign(.)$ the classic sign function. This controller can be easily implemented in hardware (bang-bang control): only sign is needed using Arduino and operational amplifiers.

Matlab/Simulink simulations using the model (1) controlled by (3) will be presented. Several possibilities: stable/unstable rotations and even asymptotic stability will be analyzed along with conclusions and future work.

Keywords: Parametric pendulum, Singular optimal control, Energy harvesting

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NONLINEAR DYNAMICS AND ENERGY HARVESTING PERFORMANCE OF SNAP THROUGH OSCILLATOR WITH TIME VARYING POTENTIAL WELL

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ABSTRACT

Conventional bistable energy harvesting systems have the limitation that for low amplitude excitation, the motion is limited to intrawell motion. The shape of the potential function have significant effect on the performance of the system. Most of the cases, the shape of the potential function is fixed and researchers have looked into the option to enhance the performance of the bistable systems by providing additional elastic constraints or enhance the magnitude of the excitation by elastic magnifiers [1]. Kim et al. [2] investigated a hybrid bistable vibration energy harvester with an adaptive potential well. The potential barrier is reduced by providing additional spring and it is observed that significant improvement in energy harvesting for low excitation amplitude. In this paper, a harmonically excited snap through vibration energy harvester is discussed whose potential function change with respect to time and can be manipulated effectively to produce interwell periodic motion. Model of the system is shown in Fig.1(a). The non-dimensional equations of motion for the system is a differential algebraic equation and is given below.

$$\ddot{x} + 2\zeta\dot{x} + x + rx\left(1 - \frac{1}{\sqrt{x^2 + \alpha(t)^2}}\right) + \theta I = f\cos(\Omega\tau) \tag{1}$$

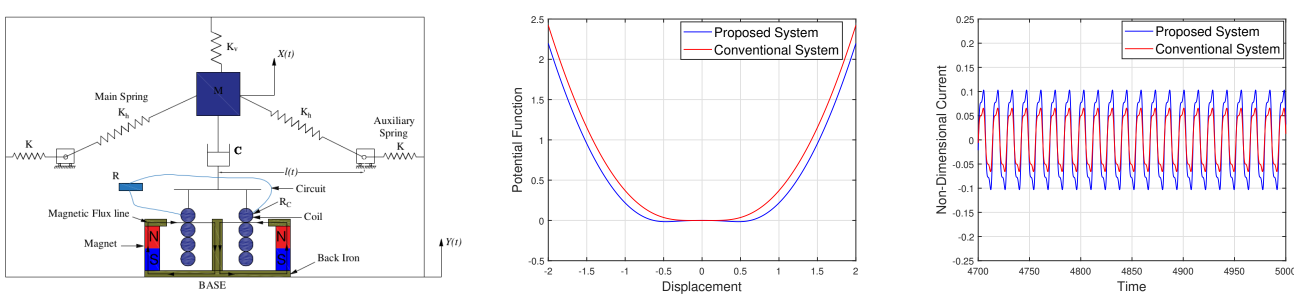
$$\dot{I} + \lambda I - \mu\dot{x} = 0 \tag{2}$$

$$\alpha(t)\left(1 - \frac{1}{\sqrt{x^2 + \alpha(t)^2}}\right) - \gamma\left(\beta - \alpha(t)\right) = 0 \tag{3}$$

Where the non-dimensional parameters are given by

$$\alpha(t) = \frac{l(t)}{L},\quad \beta = \frac{l_0}{L},\quad r = \frac{\omega_2^2}{\omega_1^2},\quad 2\zeta = \frac{C}{M\omega_1},\quad f = \frac{F}{ML\omega_1^2},\quad I = \frac{\bar{I}}{L},\quad \theta = \frac{BL_c}{M\omega_1^2},\quad \lambda = \frac{R}{L_i},\quad \mu = \frac{BL_c\omega_1}{L_i}$$

Nonlinear dynamics of the above system is investigated numerically. The motion can be intrawell/interwell periodic or chaotic. It is found from preliminary results that for certain parameter values even for low excitation, the periodic interwell motion is obtained and energy harvesting capability is enhanced. Typical potential function plot and current time history for a case with (r = 1.00, ζ = 0.015, f = 0.10, Ω = 0.45, θ = 0.40, λ = 0.80, μ = 0.30, γ = 2.50 and β = 0.25) are shown in Figs.1(b)-(c). A computational framework based on harmonic balance method is developed to obtain the periodic solutions.



(a) Model of energy harvester (b) Potential function (c) Time history

Figure 1: Dynamics of snap through energy harvester with time varying potential function

Keywords: Bistable energy harvester, Time varying potential well and Differential Algebraic Equations

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FLEXURAL-FLEXURAL THREE-TO-ONE INTERNAL RESONANCES IN HINGED-SIMPLY SUPPORTED BEAMS WITH AN AXIAL SPRING AT THE END

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ABSTRACT

Studies on planar hinged-simply supported beams show that flexural linear natural frequencies ω_{0n} are independent of boundary conditions in the axial direction, which can be modified e.g. by an axial spring k_s [1] or a tip mass M_t [2]. For slender beams, longitudinal modes are well separated and located far away from the first fundamental transverse resonances, and flexible support can only slightly modify the axial natural frequencies ω_{A0m} and corresponding linear modes. A significant reduction of the beam length, requires considering the beam model with Timoshenko shearing effects, and moves the first natural frequency ω_{A01} very close the first few bending resonances. In both cases of high and low slenderness, large amplitudes of oscillations of the system stimulate nonlinear coupling between the nth bending and the mth longitudinal modes through geometric nonlinearities and inertia interactions [3]. Coupled longitudinal-transversal vibrations of the structure are susceptible to change by varying boundary conditions (to be herein considered as a passive control) via elastic support/tip mass and, as consequence, the nonlinear response can vary from hardening to softening [1–5]. Another interesting phenomenon in nonlinear dynamics of planar beams is the internal resonance. It occurs, for example, when flexular and longitudinal natural frequencies are close to be in ratio 1 to 2 ($\omega_{0n} \approx 0.5 \times \omega_{A0m}$) [6].

In the present study we investigate another interaction between nonlinear modes, namely when the ratio of frequencies of two successive *nonlinear* flexural modes nth and $(n+1)$th are in vicinity of the ratio 1 to 3. The nth and $(n+1)$th natural frequencies are fixed for a given beams' properties and nonlinear response is controlled by a change of the axial spring stiffness k_s. The multiple time scales method is implemented directly to the partial differential equations of motion of extensible-shearable beam in *three-to-one* internal resonance condition as in [7]. Such approach enables extended description of the hardening/softening behaviour for the structure and additionally improves the quality of the frequency response curves e.g. when frequency of excitation is not only slightly distant from nth flexural mode; enables detection of more than two stable solution paths for selected frequency of excitation; provides stability information of large amplitude vibrations. Selected results are compared with numerical finite element simulations obtained by the commercial software Abaqus_CAE®.

Keywords: Beam-axial spring system, Nonlinear modes interactions, Multiple time scales method,

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